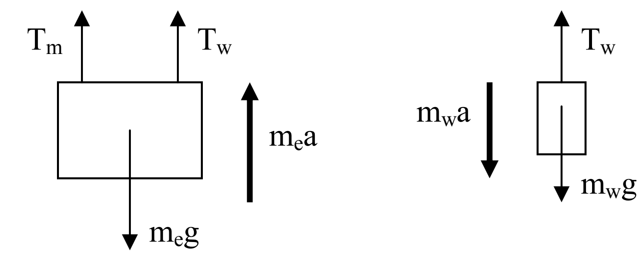
Engineering Dynamics
Homework 2 Notes

1.
In this problem a couple free body diagrams can go a long way:

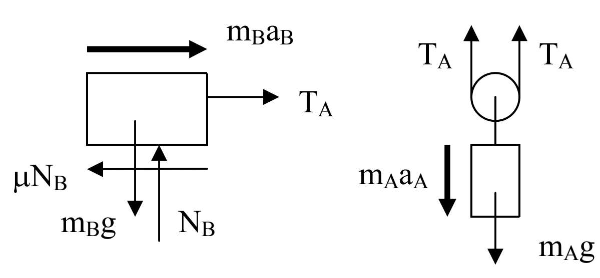


The sum of forces on the elevator result in the mass of the elevator times its acceleration. Same with the counterweight. Note, though, that the tension T_w is the same for each, and that the acceleration, though in opposite directions, is of the same magnitude. To make things easier, the acceleration vector is positive in the directions indicated by the arrows.
For the elevator:
$m_e a = \Sigma F$
$m_e a = T_m + T_w - m_e g$

For the counterweight:
$m_w a = m_w g - T_w$

Two equations, two unknowns; the acceleration being the one we care about. To solve for the velocity you can use any set of equations, but I personally prefer using
$v^2 = v_o^2 + 2a(x - x_o)$.

2.
As in the previous problem, a set of free body diagrams can help a lot:



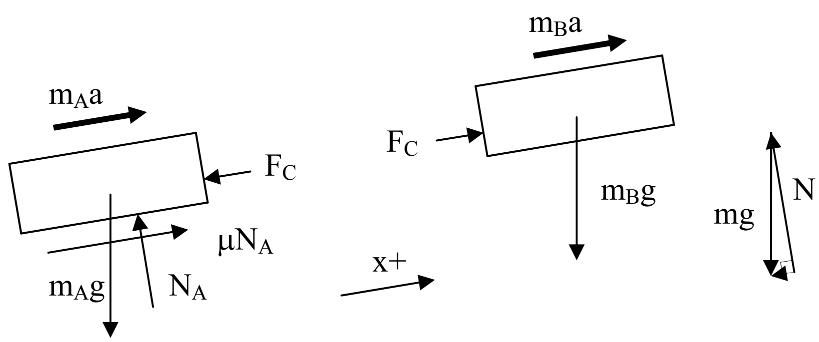
For block B:
$m_B a_B = \Sigma F$
$m_B a_B = T_A - \mu N_B$
where, by observation, we can see that $N_B = m_B g$. (N_B is the normal/reaction force supporting block B.) We have two unknowns.

For block A:

$m_A a_A = \Sigma F$
$m_A a_A = m_A g - 2T_A$

Normally, we would have a real problem, but we know that the acceleration of block A is ½ that of block B because of the pulley. We also have to note that the two cables on the pulley supporting block A result in an upward force of $2T_A$. From these we can solve for a_A, and solve for the velocity where $v = v_o + at$.

3.
A few free body diagrams…



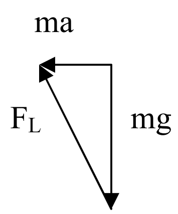
We can do this a couple ways. One is to write the equation(s) for each car. Note that the positive *x* direction is to the upper right by 5 degrees, and that the coupling force pushes against each car.
Car A:
$m_A a = -F_c + \mu N_A - m_A g \sin 5°$
$N_A = m_A g \cos 5^o$

Car B:
$m_B a = F_c - m_B g \sin 5$

The good news is that the acceleration for each car is the same. Two equations, two unknowns. Solve for the acceleration a, and plow that back into one of the others for F_c, or use a method of simultaneous equations. What ever works for you. (Trick part of this problem: The *weight* of the cars is given, not the masses.)

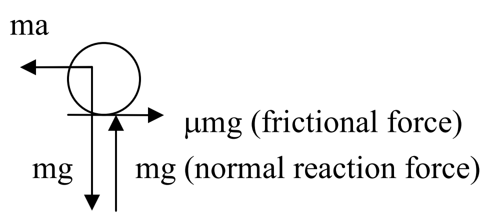
Centripetal acceleration is commonly described as $a = \dot{\theta}^2 r = \dfrac{v^2}{r}$.

4.
A free body diagram…



The acceleration, *a*, is calculated using the given radius and velocity. Use the Pythagorean theorem to calculate F_L, and an arc tangent to determine the angle.

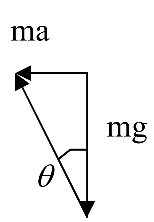
5.
A free body diagram…



Summing the forces in the (presumed) *x* direction, we see that
$ma = \mu mg$
We can drop the mass, *m*.
$a = \mu g$
The acceleration, *a*, is substituted where $a = \frac{v^2}{r}$.

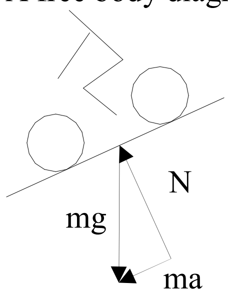
$\frac{v^2}{r} = \mu g$
Solve for *v*.

6.
A free body diagram…



The radius is $r = 4 + 6 \sin\theta$. The vertical force is the same as the weight of the rider ($F_b = mg$) . The normal force is *ma* where $a = \frac{v^2}{r}$. There is no tangential force.

7.
A free body diagram…

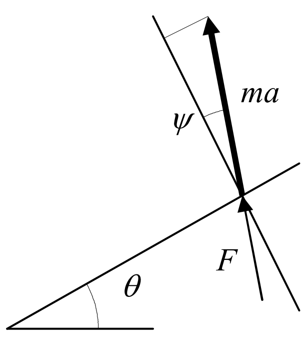


The relationship between the normal force, *N*, and the downward force due to gravity, *ma*, is the slope. The slope can be determined by taking the derivative of the curve, and solving for that particular position given *x*. So you may have to break out your calc text to get the derivative of $.2e^x$. BTW, $\dot{v}$ is the same as the acceleration, *a*.

8.
Where
$r = 1.7\cos\theta$
$\dot{r} = -1.7\dot{\theta}\sin\theta$
$\ddot{r} = -1.7\ddot{\theta}\sin\theta - 1.7\dot{\theta}^2\cos\theta$
But $\ddot{\theta}$ is presumed to be 0 because $\dot{\theta}$ is constant, so
$\ddot{r} = -1.7\dot{\theta}^2\cos\theta$

$N_r = m\ddot{r}$
$N_k = m\dot{\theta}^2 r$

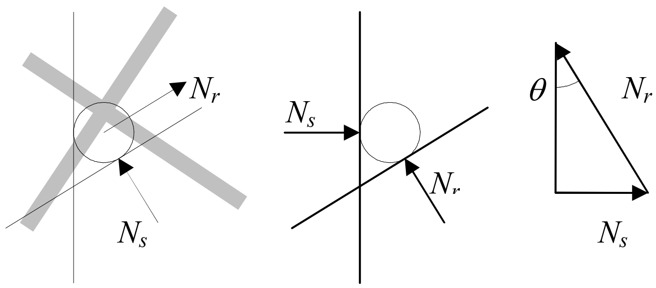
9.
A free body diagram…



The angle they want you to use, ψ, is the slope of the curve relative to a radial line from the center of rotation as illustrated above. Thus, to get that angle, you need the slope, which can be derived from the equation for the radius:
$r = .2e^{0.1\theta}$
$$\frac{dr}{d\theta} = .2e^{0.1\theta}$$

Given θ, we can solve for the angle (if we really want to):
$\psi = tan^{-1}(.2e^{0.1\theta})$ where $\theta = \pi/3$.
The applied force, F, gives us the acceleration along the tube.
$F = ma$
From the free body diagram, the tangential acceleration can be determined using the relationship
$ma_t = ma \cos \psi$
$a_t = a \cos \psi$

10.
The most difficult part of this problem is just figuring out what forces are desired and their directions. The free body diagram makes it more obvious. The labeling in the problem can be confusing, so read carefully.



The math for this problem is actually pretty straight forward:
$N_r = m\dot{\theta}^2 r$
$N_s = N_r \sin \theta$